# Nonlocal de Broglie wavelength of a two-particle system

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We show that it is possible to associate a de Broglie wavelength with a composite system even when the constituent particles are separated spatially. The *nonlocal* de Broglie wavelength ($\lambda/2$) of a two-photon system separated spatially is measured with an appropriate detection system. The two-photon system is prepared in an entangled state in space-momentum variables. We propose that the same result can be obtained for a system of massive particles separated spatially in an entangled state in space-momentum variables.

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## I. INTRODUCTION

It is well known that a de Broglie wavelength can be associated not only with single particles, but also with a multiparticle system. For a system of $N$ identical particles, the resulting wavelength is given by $\lambda_{dB}=\lambda_i/N$, where $\lambda_i$ is the de Broglie wavelength associated with the individual constituent particles [1]. Normally, these particles are held together by some kind of binding force as in the experiments done with molecules by Bordé *et al.* [2] and by Chapman *et al.* [3].

In a recent paper, Fonseca, Monken, and Pádua [4], adapting an original proposal by Jacobson *et al.* [5], measured the de Broglie wavelength of a two-photon wave packet for which the role of binding is played by entanglement. A Young interference pattern of two-photon wave packets, which behaved like single entities with twice the energy of each constituent photon, was detected. The corresponding wavelength was, of course, half the de Broglie wavelength of a single photon. Although in all measurements there were two photons in the same wave packet, it was shown in [4] that the measured de Broglie wavelength $\lambda$ or $\lambda/2$ depends on the two-photon state. Two or more photons in the same wave packet do not necessarily interfere as a ''bound system.'' The multiparticle de Broglie wavelength is measured only if the composite system as a whole interferes with itself for the formation of the interference pattern. This will depend on its state at the entrance of the interferometer, on the interferometer one uses, and on the detection system.

At this point an interesting question arises: is it possible to associate (and to measure) a de Broglie wavelength with a system of macroscopically separated free particles? In this article we show that, at least for photons, the answer is *yes*. The experimental results suggest that entanglement is a sufficient condition to define a de Broglie wavelength for a multiparticle system. We further propose that the same is true for entangled massive particles separated spatially. As long as they are in an entangled state in space-momentum variables, the de Broglie wavelength of the system can be measured with an interferometer analogous to the one described here.

An entangled two-photon field can be generated by spontaneous parametric down-conversion (SPDC). In the process of SPDC, a pump photon incident upon a nonlinear crystal splits into a pair of photons, usually called the signal and idler [6]. There have been numerous interesting two-photon interference experiments (see, for example, [4,7–19]). In this paper we focus on an interesting ''nonlocal'' aspect of entanglement. By modifying the transverse field profile of the pump laser beam in the SPDC process and manipulating the detection system we obtained a fourth-order interference pattern of the down-converted photons with periodicity $\lambda/2$ when the photon pairs are transmitted by two Young double slits. Therefore, we are able to measure the de Broglie wavelength of the two-photon system even when the constituent photons are separated spatially. Another interesting result is that, depending on the detection system, no fourth-order interference pattern is observed. All these results are predicted by a quantum multimode calculation [16,20,21].

In Ref. [12] it was shown that the angular spectrum of the pump beam is transferred to the two-photon state generated by SPDC. As a consequence, the probability distribution for two-photon detection $P_2(x_s,x_i)$ reproduces the transverse pump intensity profile $W(x)$, in the following way:

$$P_2(x_s,x_i)\propto W\left(\frac{x_s}{\mu_s}+\frac{x_i}{\mu_i}\right), \tag{1}$$

where $\mu_s=k_p/k_s$, $\mu_i=k_p/k_i$, and $k_p$, $k_s$, $k_i$ are the wave numbers of the pump, signal, and idler fields, respectively. Regarding the photons as particles traveling with the same velocity, the above expression means that it is possible to control the transverse coordinates of their ''center of mass'' via the pump beam profile.

Consider that signal and idler photons are incident on two double slits as shown in Fig. 1. Let us discuss only the cases in which both photons are transmitted by the slits. By focusing the pump beam on $x=0$, it is possible to force the pair to go through either slits $A_s^a,A_i^b$ or $A_s^b,A_i^a$ [Fig. 1(a)]. On the other hand, by creating a pump beam profile peaked at both $x=+d$ and $x=-d$ (see below), it is possible to force the

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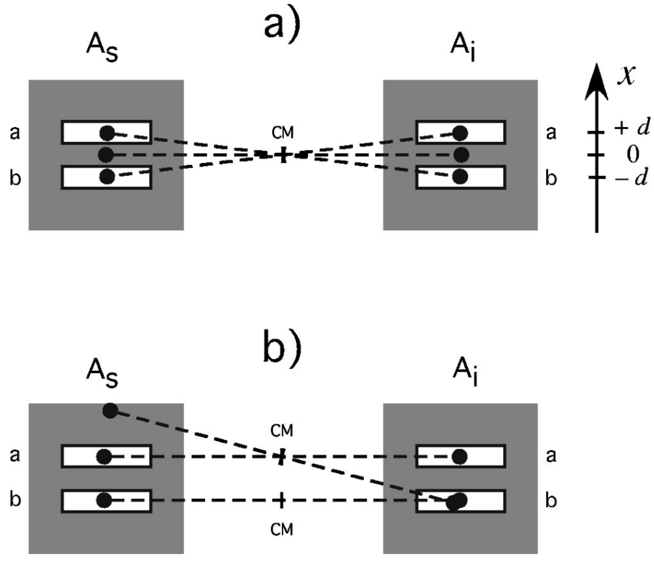


FIG. 1. Double-slit arrangements used to produce Young-type interference with pairs of particles. The fourth-order spatial correlation is focused (a) on $x_s+x_i=0$; (b) on both $x_s/2+x_i/2=-d$ and $x_s/2+x_i/2=+d$. For massive particles, this is equivalent to focusing their center of mass on (a) $x=0$ and (b) $x=\pm d$. The diagonal line shown in (b) indicates the case where one of the particles is transmitted and the other is absorbed by the double-slit screen.

pair to go through either slits $A_s^a, A_i^a$ or $A_s^b, A_i^b$ [Fig. 1(b)]. If the two photons are detected on a distant plane, there are, in each case, two indistinguishable paths leading to a coincidence detection. Then one should expect to see Young-type interference of the photon pair with itself when the detectors are moved. We measured this interference in our experiment.

## II. EXPERIMENT

The experimental setup used is sketched in Fig. 2. A 5 mm×5 mm×7 mm BBO (beta barium borate) nonlinear crystal pumped by a 200 mW krypton laser emitting at 413 nm was used to generate type II SPDC. Down-converted photons with a degenerate wavelength $\lambda=826$ nm propagating at angles of 5° with the pump laser beam direction were selected. Two identical Young double slits ($A_i$ and $A_s$) are placed at the exit path of the signal and idler beams at the same distance of 455 mm from the crystal (Fig. 2). The width of each slit and the distance between them are $2a=0.072$ mm and $2d=0.26$ mm, respectively. The double-slit planes (approximately in the $xy$ plane) are aligned perpendicular to the plane defined by the pump laser and the down-converted beams ($yz$ plane) with the small dimension of the slits parallel to the $x$ direction (Fig. 2). Detectors $D_i$ (idler) and $D_s$ (signal) detect coincidences between the idler and signal photons transmitted through the two double slits. Light detectors are avalanche photodiodes, placed at a distance $z_1=1060$ mm from the crystal. An arrangement composed of a single collimating slit of width $2b=0.20$ mm oriented parallel to the Young slits, followed by a microscope objective lens, is placed in front of each detector. $D_i$ and $D_s$ are connected to single and coincidence counters, with a coincidence detection resolving time of 5 ns.

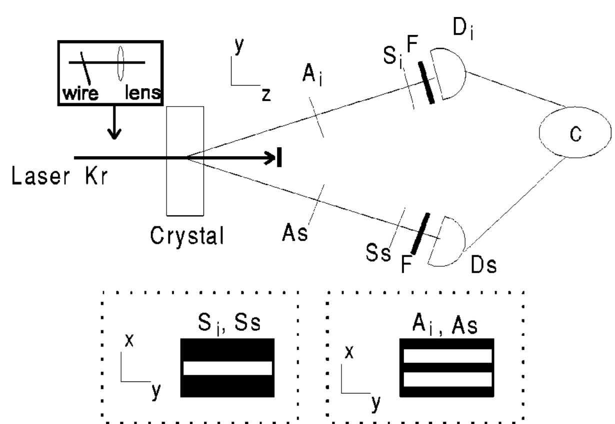


FIG. 2. Outline of the experimental setup. Photon pairs are generated by SPDC (see text). $A_i$ and $A_s$ are two double slits; $F$ is an interference filter; $S_i$ and $S_s$ are single slits; $D_i$ and $D_s$ are detectors that move in the $x$ direction; and $C$ is a coincidence counter. Each double slit is formed by the two identical slits $a$ and $b$. (Inset) In some of the measurements a wire and a lens were positioned in the pump laser beam path.

## III. RESULTS

Fourth-order interference patterns were obtained for two different transverse pump beam profiles. The arrangement sketched in Fig. 1(a) was implemented by focusing the pump beam 460 mm after the crystal. We used a 500 mm focal length lens, placed 40 mm before the crystal. The second arrangement [Fig. 1(b)] was implemented by projecting the shadow of a wire 460 mm after the crystal so as to create an intensity profile with two peaks, one close to $x=+d$ and the other close to $x=-d$. A 0.125 mm diameter steel wire was aligned parallel to the Young double slits and placed 1540 mm before the crystal. A 500 mm focal length lens was placed 540 mm before the crystal (Fig. 2 inset). The pump beam transverse profiles in the $x$ direction, measured at $z=460$ mm after the crystal, can be seen in Fig. 3. The profiles were measured by displacing a 0.015 mm diameter pinhole transversely to the beam and measuring with a power meter the transmitted laser intensity as a function of the pinhole position. The Gaussian transverse beam profile with a measured 0.068 mm full width at half maximum (FWHM) is presented in Fig. 3(a). In Fig. 3(b) we see the two-peak measured intensity profile created by the shadow of the 0.125 mm diameter wire.

Working in the degenerate case ($k_s=k_i=k$) and in the Fraunhofer regime, the number of coincident photons at positions $x_s$ and $x_i$ for the first case (pump beam focused on $x=0$) can be approximated by [16]

$$N_c(x_s,x_i)=F(x_s,x_i)\left\{1+\cos\left[k(x_s-x_i)\frac{2d}{z_1}\right]\right\}, \qquad (2)$$

where $z_1$ is the distance between the double slits and the detectors. In the second case (pump beam with peaks at $x=+d$ and $x=-d$), $N_c$ can be approximated by [16]

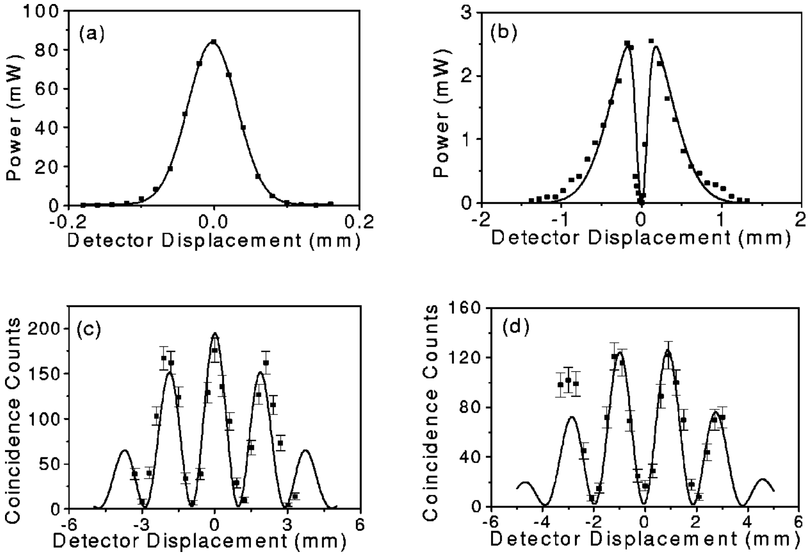


FIG. 3. (a) Transverse profile of the laser beam measured at $z_A = 455$ mm from the crystal when the laser beam is focused at $z_A$. (b) Transverse profile of the pump laser at $z_A = 455$ mm from the crystal when the wire and lens were positioned in the beam path. The continuous curves are Gaussian fits. In (c) and (d), for the transverse pump profile shown in (a), fourth-order interference fringes were measured by scanning the idler detector and by letting the signal detector be fixed at the positions $x_s = 0.0$ mm and $x_s = 1.0$ mm, respectively.

$$N_c(x_s, x_i) = G(x_s, x_i)\left\{1 + \cos\left[k(x_s + x_i)\frac{2d}{z_1}\right]\right\}, \qquad (3)$$

where $F(x_s, x_i)$ and $G(x_s, x_i)$ contain diffraction terms. Both expressions (2) and (3) show that we can obtain fringes with a periodicity corresponding to a two-photon de Broglie wavelength by scanning simultaneously both signal and idler detectors. Our results are presented in Figs. 3(c) and 3(d) and Fig. 4. In Figs. 3(c) and 3(d), we observe fourth-order interference patterns scanning the position of detector $D_i$ while keeping $D_s$ fixed at positions $x_s = 0.0$ mm [3(c)] and $x_s = 1.0$ mm [3(d)], for the Gaussian pump profile of Fig. 3(a). A similar conditional interference pattern is obtained for the pump profile of Fig. 3(b) [16].

If the transverse pump profile is the one shown in Fig. 3(a), a fourth-order interference pattern with doubled periodicity is obtained by scanning simultaneously the idler and signal detectors in opposite directions with the same step ($+x_i$ and $-x_s$, respectively). This is shown in Fig. 4(a). However, when we move $D_i$ and $D_s$ in the same direction ($+x_i$ and $+x_s$ directions) with the same step, no interference pattern is observed [Fig. 4(b)]. For the transverse pump profile shown in Fig. 3(b) the results are the opposite. No interference pattern [Fig. 4(c)] occurs when we displace the detectors in opposite directions. The Young interference pattern [Fig. 4(d)] has a period proportional $\lambda/2$ when the two detectors are scanned simultaneously in the same direction. Physically, the control of the fourth-order spatial correlation through the transverse intensity profile of the pump laser [12,18] makes possible the visualization of the pure two-photon effects shown in Fig. 4. Also, by manipulating the detection system, different effects are seen: a fourth-order interference pattern with periodicity proportional to $\lambda/2$, as well as a situation where we observe no interference pattern at all.

Solid curves in Fig. 4 are theoretical curves with one normalization parameter [16]. The transverse pump profiles at the double-slit position $W(x, z_A)$ are obtained from the fits of the experimental data (Fig. 3). It is important to point out that the Young's interference pattern shown in Fig. 4(a) pre-

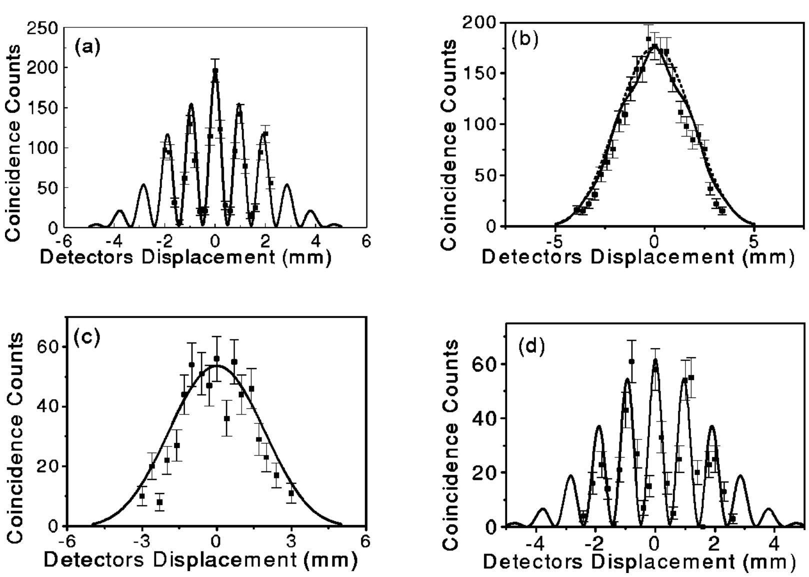


FIG. 4. For the pump beam profile shown in Fig. 3(a), fourth-order interference fringes are shown in (a) and (b); for the pump profile of Fig. 3(b), they are shown in (c) and (d). (a) and (b) show the coincidence counts as a function of the simultaneous displacement of the detectors in opposite directions ($+x_i$ and $-x_s$ directions) and in the same direction ($+x_i$ and $+x_s$ directions), respectively. Coincidence count detection time was 200 s. (c) and (d) show the coincidence counts as a function of the simultaneous displacement of the detectors in opposite directions ($+x_i$ and $-x_s$ directions) and in the same direction ($+x_i$ and $+x_s$ directions), respectively. Coincidence count detection time was 1000 s.

sents the same characteristics as the one shown in Fig. 4(d), although they were obtained with different detection procedures. The same occurs in Figs. 4(b) and 4(c). The interference pattern in Fig. 4(b) has a very small visibility, but not zero. This is due to the finite FWHM of the transverse pump profile [Fig. 3(a)]. Calculation shows that, by narrowing the pump profile even further, the down-converted photon correlation is maximized [17,21]. The dashed curve shows the expected result when the transverse pump profile is a spatial $\delta$ function.

## IV. DISCUSSION AND CONCLUSION

Our results can be understood in terms of the physical picture described in Fig. 1. For the pump profile of Fig. 3(a), the spatial correlation of the generated photon pairs corresponds to the situation depicted in Fig. 1(a). The interfering pathways described in Fig. 1(b) correspond to the pump profile of Fig. 3(b). We have checked this experimentally by detecting the transverse profiles of the twin photons in coincidence at the positions of the double slits [21]. We can also understand the results of Figs. 4(b) and 4(c). In order to properly define a nonlocal de Broglie wavelength, one minimal requirement is that lengths be defined the same way at each ''measuring apparatus.'' In the measurements where we see no interference, lengths are defined in opposite directions. For a local system this would be analogous to displacing the detector and then bringing it back to its original position with no net displacement. If one does not maintain the same length definition for observers located at detectors $D_s$ and $D_i$, any fringe periodicity can be, in principle, observed [22].

In [4], photon pairs were generated collinearly and the fourth-order interference pattern was recorded by displacing the entire ''two-photon detector'' transversely to the double-slit plane [4,15]. The two-photon detector collects only those photon pairs that fall in the same spatial region defined by its entrance slit. We regard it as a ''local detection system.'' In the present experiment we measure the de Broglie wavelength of the biphoton with a ''nonlocal detection system,'' since the photons belonging to the same pair fall on different spatial regions defined by the slits of each detector ($D_i$ and $D_s$). In [4], the entangled photon pairs interfere like a *local single entity*. In this work the biphoton interferes like a *nonlocal single entity*. In both cases we measured the de Broglie wavelength of the biphoton: $\lambda/2$.

Our experimental results show that it is possible to define and to measure the de Broglie wavelength of a system of macroscopically separated photons when they are generated in an entangled state in momentum-space variables. The same should be true for massive particles. In a recent experiment, Sackett *et al.* [23] produced entanglement between two and four trapped ions and observed interference fringes with a periodicity proportional to the number of entangled particles (which are not macroscopically separated in the sense we mean here). There are other possible experiments one can think of with massive particles. For example, we could consider resonant photodissociation of a beam of diatomic molecules. Fry and co-workers have proposed to photodissociate $Hg_2$ dimers for the generation of a two-atom ''spatially separated entangled state'' [24]. Since transverse momentum is conserved, it may be possible by using this method to produce a two-atom entangled state in momentum-space variables. This can be demonstrated in a simple experiment, similar to the one we performed with photons (with double-slit interferometers). Let us suppose for simplicity that the two atoms have equal masses $m$ and propagate with equal longitudinal momenta $p_j=\hbar k_j=\hbar k$ ($j=1,2$) which are large compared to the transverse momentum changes during the course of the experiment. This allows us to use the paraxial approximation. We also assume that $k_1=k_2=k$ and that the slits are infinitesimally wide. Due to transverse momentum conservation, the two atoms arrive at the double slits [25] in the way described above: the atom pair goes through slits $A_s^a, A_i^b$ or $A_s^b, A_i^a$ [Fig. 1(a)]. Therefore, the two-atom state immediately after the double slits is

$$|\Psi\rangle=\frac{1}{\sqrt{2}}\{|d\rangle_1|-d\rangle_2+|-d\rangle_1|d\rangle_2\}, \tag{4}$$

where $2d$ is the separation of the double slits. Following a similar calculation done by Walls and Milburn [26] we obtain the probability density of detecting the first and the second atoms at the transverse positions $x_1$ and $x_2$ after the respective double slits,

$$|\langle x|\Psi\rangle|^2\propto 1+\cos\left[k(x_1-x_2)\frac{2d}{z_1}\right], \tag{5}$$

where $z_1$ is the distance between the double slits and the atom detectors' plane. On the other hand, if we dissociate the dimers in such a way that by momentum conservation the atom pair goes through either slits $A_s^a, A_i^a$ or $A_s^b, A_i^b$ [as in Fig. 1(b)], where the center of mass is focused on $x=\pm d$], the two-atom state after the double slits is

$$|\Psi\rangle=\frac{1}{\sqrt{2}}\{|d\rangle_1|d\rangle_2+|-d\rangle_1|-d\rangle_2\}, \tag{6}$$

and the probability density of detecting the first and the second atoms at the transverse positions $x_1$ and $x_2$ after the respective double slits is,

$$|\langle x|\Psi\rangle|^2\propto 1+\cos\left[k(x_1+x_2)\frac{2d}{z_1}\right]. \tag{7}$$

It can be seen immediately that, if we detect the atom pairs in coincidence by displacing the detectors in opposite directions for the first case or in the same direction for the second case, we are able to measure a de Broglie wavelength associated with the atom pair, i.e., $\lambda_{dB}=\lambda/2$, as for the down-converted photons. Again, this de Broglie wavelength is associated with a spatially separated atom pair in an entangled state and the atom pairs interfere as a nonlocal single entity.

Entanglement between pairs and, more recently, triplets of (moving) atoms has also been demonstrated in the framework of cavity QED [27,28]. Those results can be adapted to produce entanglement in momentum-space variables and it should be possible to conceive an experiment similar to the photonic case. The concept of a nonlocal de Broglie wavelength introduced here is a generalization of the de Broglie wavelength associated with a multiparticle system. We showed that it is not necessary for the particles to be physically bound together or even localized: *entanglement* is a sufficient ingredient.

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